Study of the Strongly Interacting Higgs Physics Model with Heavy Fermions

L.V. Dung[1], G. Jikia[2], C. Roiesnel and T.N. Truong

Centre de Physique Théorique,

Centre National de la Recherche Scientifique, UPR A0014,

Ecole Polytechnique, 91128 Palaiseau Cedex, France

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Abstract

We study the effect of a possible fourth heavy generation of fermions on the Higgs sector of the standard model. We show, from the violation of elastic unitarity, that the scale of strong interactions is well below 1 TeV even with a Higgs mass as low as 500 GeV provided the fourth generation fermion mass is equal or larger than the Higgs mass. The diagonal Pade approximant method is then used to unitarize the partial wave amplitudes. It is found that, for the fourth generation fermion masses which are comparable to or larger than the Higgs mass, the Landau ghosts in the I=0 and I=2 channels of the reconstructed amplitudes move too close to the physical region to be accepted.

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[1]Permanent address: *Theoretical Physics Institute, BP429 BoHo, Hanoi 10000, Vietnam.*

[2]Permanent address: *Institute for High Energy Physics, 142284, Protvino, Moscow Region, Russia.*

1   Introduction

The standard model is verified to a great accuracy by recent experiments at LEP. The symmetry breaking of the electroweak interaction is however less understood because its effect is insensitive to these low energy experiments. This is the consequence of Veltman's screening theorem [1]. Next generation experiments at LHC are designed to probe the Higgs sector. If the Higgs boson is sufficiently heavy, the study of the longitudinal vector boson scattering can provide valuable informations on the spontaneous symmetry breaking mechanism of the standard model. The theoretical study of this process is a challenge to theoreticians because the standard perturbative field theoretical methods have to be modified due to the strong interaction of the heavy Higgs sector. This problem is well-known and has recently been the focus of extensive studies in the litterature [2] based on the equivalence theorem [3] with the linear sigma model [4].

An equally interesting problem, but less well-known, is to consider the effect of a possible heavy fourth generation of fermions [5]. The existence of heavy weak $SU(2)$ multiplets, sufficiently degenerate to satisfy the ρ parameter constraint, is still consistent with experimental data. Heavy fermions interact strongly because of large Yukawa couplings and along with the Higgs boson represent strongly interacting quanta of the symmetry breaking sector. This hypothesis will introduce another scale in the problem and physical amplitudes, even at low energy, does not obey the screening theorem, i.e. the Higgs and fermion mass dependence of physical quantities is no longer logarithmic but rather power like. This result is related to Hill's conjecture [6] about the non-decoupling of heavy quanta when adding extra strong interactions in the Higgs sector. In a previous preliminary study of the consequences of the existence of a fourth generation of fermions [7], it was found that the P-wave elastic scattering amplitude of the longitudinal W bosons depends sensitively on the ratio of the Higgs and fermion masses. The purpose of this letter is to make a more general and precise study of this question.

Our strategy is to calculate numerically the one-loop $W_L W_L$ partial-wave amplitudes in all isospin channels. Using the Argand diagram analysis we show, from the violation of elastic unitarity, that the strong interaction physics sets in even with a Higgs mass as low as 500 GeV provided the 4th generation fermion mass is equal or larger than the Higgs mass. We then make the hypothesis that the elastic unitarity condition at low energy, where inelasticity is negligible, is an important condition which must be satisfied by strong interactions. This leads us to unitarize the pertubative amplitudes by the Padé approximant method [8]. We also require that the unitarisation scheme should be consistent with analyticity or causality or, in other words, the presence of the Landau ghosts due to the unitarisation scheme should not be accepted if they are too close to the low energy region. We study this Landau ghost problem using the low energy expansion of the one loop amplitude and find that the solutions with the fourth generation fermion masses which are comparable or larger than the Higgs mass,

the Landau ghosts in the $I = 0$ and $I = 2$ channels move too close to the physical region to be accepted.

Our main conclusion is that if the ratio of the Higgs to the fermion mass is less or equal to unity, our requirement cannot be satisfied with any known non perturbative methods. Large values of the ratio of Higgs to fermion mass are entirely acceptable. Although we work with a heavy Higgs assumption so that the equivalence theorem can be applied, which simplifies considerably our calculation, we suspect that this difficulty is also present in a scenario with a relatively light Higgs of a few hundred GeV but with heavier fermions.

2   Argand plots for $W_L W_L$ scattering

The real parts of the one-loop corrections to longitudinal vector boson scattering due to a heavy fourth generation of fermions have been calculated by Dawson and Valencia [5] using the Goldstone boson equivalence theorem [3] and the On-Mass-Shell renormalization scheme. Their calculation can be shown to be in agreement with a much earlier one by Jhung and Willey [9]. For a numerical evaluation by the FF package [10], we find it convenient to express the complex $W_L W_L$ scattering amplitudes in terms of the 4-, 3- and 2-point scalar loop integrals. When the fermions are degenerate, we need only to calculate the one-loop amplitude $A(s, t, u)$ for the process $W_L^+ W_L^- \to Z_L Z_L$. We get

$$A(s, t, u) = A_b + A_f$$

where A_b and A_f, respectively the bosonic and fermionic contribution, read

$$
16\pi^2 v^4 \, A_b = m_H^8 \Big\{ D_{wHwH}(t, s) + D_{wHwH}(u, s) \Big\} + 2m_H^6 \Big\{ C_{wHw}(t) + 2\, C_{wHw}(u) \Big\}
$$

$$
+ \frac{2\, m_H^6}{s - m_H^2} \Big\{ s\, C_{wHw}(s) + \big(s + 2\, m_H^2\big)\, C_{HwH}(s) + 2\, B_{wH}(0) + s\, B'_{wH}(0) \Big\}
$$

$$
+ \frac{m_H^4}{2\,(s - m_H^2)^2} \Big\{ s\,\big(7s - 4m_H^2\big)\, B_{ww}(s) - 3s^2\, \Re e B_{ww}(m_H^2) - 9s^2 B_{HH}(m_H^2)
$$

$$
+ \big(s + 2\, m_H^2\big)^2\, B_{HH}(s) \Big\} + m_H^4 \Big\{ B_{ww}(t) + B_{ww}(u) \Big\} - \frac{m_H^4 s}{2\,(s - m_H^2)} \tag{1}
$$

$$
\frac{16\pi^2 v^4}{N_c}\, A_f = 4\, M_f^4 \Big\{ st\, D_{ffff}(s, t) + su\, D_{ffff}(s, u) - tu\, D_{ffff}(t, u) \Big\}
$$

$$
- \frac{16\, M_f^4 s^2}{(s - m_H^2)}\, C_{fff}(s) - \frac{4\, M_f^2}{(s - m_H^2)^2} \Big\{ \big(m_H^2 - 4\, M_f^2\big)\, s^2\, \Re e B_{ff}(m_H^2)
$$

$$
+ 4\, M_f^2 s^2 B_{ff}(s) - m_H^4\, s\, \big(B_{ff}(s) - B_{ff}(0)\big) - m_H^2\, s^2\, B_{ff}(0) \Big\} \tag{2}
$$

3

and the scalar loop integrals, denoted as D, C and B, are defined by the standard expressions

$$D_{wHwH}(t,s) =$$
$$\frac{1}{i\pi^2} \int \frac{dQ}{Q^2((Q+p_3)^2 - m_H^2)(Q+p_2+p_3)^2((Q+p_1+p_2+p_3)^2 - m_H^2)},$$
$$C_{wHw}(s) = \frac{1}{i\pi^2} \int \frac{dQ}{Q^2((Q+p_1)^2 - m_H^2)(Q+p_1+p_2)^2},$$
$$B_{mM}(s) = -\int_0^1 dx \, \log(xm^2 + (1-x)M^2 - x(1-x)s),$$
$$B'_{mM}(s) = \frac{dB_{mM}(s)}{ds}, \quad \text{etc.},$$

with $s = (p_1 + p_2)^2$, $t = (p_2 + p_3)^2$, $u = -s - t$.

The scattering amplitudes $A_I(s,t)$ of the isospin $I = 0, 1, 2$ eigenstates are

$$A_0(s,t) = 3\,A(s,t,u) + A(t,s,u) + A(u,t,s)$$
$$A_1(s,t) = A(t,s,u) - A(u,t,s)$$
$$A_2(s,t) = A(t,s,u) + A(u,t,s)$$

and the partial-wave amplitudes $a_{IJ}(s)$ are defined by

$$a_{IJ}(s) = \frac{1}{64\pi} \int_{-1}^{+1} d(\cos\theta)\, P_J(\cos\theta)\, A_I(s, \cos\theta), \quad t = -\frac{s}{2}(1 - \cos\theta)$$

Given these exact one-loop partial-wave amplitudes, one may try to estimate, as a function of the Higgs mass m_H and heavy fermion mass m_f, the scale $\sqrt{s}$ at which the standard model becomes strongly interacting in the Higgs sector. Violation of unitarity can be a useful guide to limit the region of validity of the one-loop calculation. For that purpose, we use the Argand plots of partial-wave amplitudes to find out the degree of violation of elastic unitarity which can be ascribed to the omission of higher order terms in the pertubation series because this expansion only obeys unitarity order by order.

Let $a_{IJ}^{(0)}$ and $a_{IJ}^{(1)}$ be respectively the tree-level contribution and the one-loop corrections to the partial-wave amplitude $a_{IJ}(s)$. Then one can quantify the size of unitarity violation [11] by the vector Δa_{IJ} of minimum length which can be added to $a_{IJ} = a_{IJ}^{(0)} + a_{IJ}^{(1)}$, to bring the elastic scattering amplitude to the unitarity circle

$$|\Delta a_{IJ}| = \left| \frac{1}{2} - \sqrt{\left(a_{IJ}^{(0)} + \Re a_{IJ}^{(1)}\right)^2 + \left(\frac{1}{2} - \Im a_{IJ}^{(1)}\right)^2} \right|$$

There is some arbitrariness in choosing a criterion for satisfactory convergence of the pertubative expansion. We impose

$$\left| \frac{\Delta a_{IJ}(s)}{a_{IJ}(s)} \right| \le \frac{1}{2}$$

4

The energy scales $\Lambda_c = \sqrt{s}$ which violate this bound are gathered in Table 1 for a representative set of Higgs and fermion masses. Argand plots for $I = 0$, $J = 0$ channel are shown in Fig. 1.

Imposing other criteria such as $\left| \dfrac{a_{IJ}^{(1)}}{a_{IJ}^{(0)}} \right| < 1$, or $|\Delta a_{IJ}| < \dfrac{1}{2}$, would lower the scales Λ_c. There are no other assumptions than those which ensure the applicability of the equivalence theorem and the elastic unitarity condition, namely $M_W \ll \sqrt{s} < 2 \inf(m_H, m_f)$ [1]. The influence of the top quark with a mass smaller or equal to 200 GeV has negligible effect on the $W_L W_L$ scattering. The effect of multiple W production has recently been calculated in the heavy Higgs limit [11] and was shown to have negligible effect on the unitarity relation. In the region of energy of interest we can therefore safely ignore inelastic effects in the unitarity relation.

One can read from Table 1 that fermions induce strong interactions below 1 TeV in the $I = J = 0$ channel already for Higgs and fermion masses between 0.5 GeV and 1 GeV, whereas in the absence of fermions, interactions become strong below 2 TeV only when the Higgs mass is greater or equal to 3 TeV [11].

3   Padé approximant for $W_L W_L$ scattering

In this section we want to study the $W_L W_L$ scattering problem by resumming the one-loop calculation in order to make it satisfy the unitarity relation. This is necessary in order to extend the region of validity of the perturbation calculation as this method can handle strong interactions and/or resonance effects. The exact preservation of probability is utmost important to handle these problems.

Let us denote the lth partial wave projection of the perturbative amplitude by f where the isospin and l^{th} partial wave indices and the s dependence are omitted. The corresponding tree and one loop amplitude are denoted respectively by $f^{(0)}$ and $f^{(1)}$. The perturbative amplitude $f = f^{(0)} + f^{(1)}$ satisfies the perturbative unitarity relation, $Im f = \left(f^{(0)} \right)^2$, which is to be compared with the exact elastic unitarity relation, $Im f = (f)^2$, which must be valid in the energy region below the inelastic threshold. The one-loop perturbative series can be resummed by consructing the diagonal $[1, 1]$ Padé approximant:

$$f^{[1,1]} = \frac{f^{(0)}}{1 - f^{(1)}/f^{(0)}} \tag{3}$$

which satisfies the elastic unitarity relation $Im f^{[1,1]} = \mid f^{[1,1]} \mid^2$. Hence we can write $f^{[1,1]} = e^{i\delta} \sin \delta$, where δ is the phase shift.

If we add any real polynomial contribution to the denominator of Eq 3 or any analytic function which is real on the unitarity cut, the elastic unitarity relation

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[1]Of course the condition $\sqrt{s} < 2m_H$ is relaxed in isospin channels $I \neq 0$.

remains valid. Hence the Padé method can also be used to study the inelastic effect of the HH and FF intermediate states on the elastic scattering provided we limit ourself to the energy region below the inelastic threshold.

In Fig. 2 some typical phase shifts are plotted for Higgs boson masses $m_H = 1$ and 1.5 TeV as a function of energy. Because of our unitarisation method, for a Higgs mass less than 3 TeV, the Higgs mass defined by the position where the $I = J = 0$ phase shift passing through $90°$ is the same as that defined by the vanishing of the real part of the inverse of the Higgs propagator. For a Higgs mass larger than 3 TeV a lower energy resonance is generated with a phase shift passing through $90°$ from below and is genuinely a bonafide resonance. The Higgs mass that we defined and used in our perturbative calculation corresponds in fact to a $I = J = 0$ phase shift passing through $90°$ from above.

For $m_H^2/m_f^2 \leq 1$ the $I = 2$ S-wave phase shift also exhibits a resonance behavior but with the phase shift passing through $-90°$. We show below it is associated with a Landau ghost very close to the real energy axis and cannot be accepted. It is generated by the unitarisation scheme in which the scattering length and the effective range, respectively the $O(p^2)$ and $O(p^4)$ terms, are both negative due the contribution of the fermion loop in this channel.

The $I = J = 1$ phase shift has a resonance behavior (ρ). The position of the resonance is much lower than that given by technicolor models [7]. Furthermore the KSRF relation is not valid. Our calculation shows that for small values of Higgs mass, although the analytical results of [7] are qualitatively correct, a substantial correction to the width of ρ due to the contribution of the $t\bar{t}$ channel must be made. This is so because the ρ width due to the $W_L W_L$ states are so small that even the $t\bar{t}$ contribution becomes dominant.

4   Some problems associated with unitarisation

The unitarisation scheme does not always work because it may introduce unwanted singularities in the complex energy plane which give rise to a violation of causality. These Landau type singularities can be got rid of by multiplying the scattering amplitude by an appropiate polynomial. This can be obviously done but at the expense of a violation of unitarity. If they were far from the physical region of interest, the resulting violation of unitarity would be small and could be blamed on our approximation scheme and therefore could be accepted. Therefore we should examine carefully the unitarised amplitudes to see whether they have unwanted singularities which are too close to the physical region of interest.

However our Padé amplitude is computed numerically for real energy and is not suitable for analytic continuation to find complex singularities. A more exact calculation of the Landau ghost resulting from our Padé amplitude will be undertaken shortly and will be a subject for a future publication. In the meanwhile we can use the $s \ll m_H^2$ limit of the partial-wave amplitudes $a_{IJ}(s)$

[5], which turns out to be valid for a wide range of energy, to find the Landau ghost. The contribution of the fermion loop to these low-energy amplitudes can be summarized as follows

$$a_{IJ}^f(s) = \frac{s^2}{512\pi^3 v^4} N_c E_{IJ}(r) \,, \quad r = \frac{M_H^2}{m_f^2} \tag{4}$$

where $E_{IJ}(r)$ is a channel dependent algebraic factor and N_c is the number of color or fermion species. We shall refer loosely to $N_c E_{IJ}(r)$ as the "effective number of fermions" in the IJ channel. In the limit of the non linear sigma model (NLSM)[12], $r \to \infty$, we have $E_{00}(\infty) = 1/3$, $E_{11}(\infty) = 2/9$, $E_{20}(\infty) = 2/3$ and these numbers are all positive and small. For finite r this is no longer the case and the effective numbers of fermions, which vary wildly with the mass ratio r, can be large and negative. We find that:

$$\lim_{r \to 0} \frac{r E_{IJ}(r)}{E_{IJ}(\infty)} = -\frac{176}{3}, \; 8, \; -\frac{16}{3} \tag{5}$$

for a_{00}^f, a_{11}^f and a_{20}^f amplitudes respectively. These limits are a good approximation already when r is of order unity or less. On the contrary the NLSM limit can only be reached when r is much larger than unity. For example the value $E_{00} = 0.2$ (compared with the NLSM value of $1/3$) is reached only when $r = 40$.

The main problem with perturbation theory comes from the large effective negative number of fermions in the $I = J = 0$ channel when $r \leq 1$. Such a large negative number will inevitably introduce a Landau ghost close to origin. This is an essential point. In Table 2 we give the position of the Landau ghost as a function of fermion and Higgs masses. The result from our calculation is very unsatisfactory: As can be seen from this table, for most values of r, equal or less than unity, the Landau ghost moves towards the origin along the negative s-axis. Because our calculation should be a good low energy calculation, the presence of the low energy Landau ghost is therefore not acceptable.

The $I = 2$, $J = 0$ channel has a similar problem. For $r \leq 1$ the effective fermion number for this channel is also negative. For $r < 1$ the Landau ghost in this channel moves near the real positive s-axis. It gives rise to a "resonance" in this channel with a negative scattering length and negative effective range which is completely unphysical. It is not clear whether this undesirable solution is due to the Padé scheme or to some thing more fundamental which is related to a negative effective fermion number.

Unlike the S-wave scattering, the P-wave channel $I = J = 1$ has the effective fermion number positive for all value of r. In this case there is no Landau ghost present in our solution even with fermion and Higgs values which give difficulties in the S-wave channel.

In summary, we find that the Landau ghosts are far from the physical region of interest provided that the ratio $r \gg 1$. Although we found satisfactory solutions

for the $I = J = 1$ channel even for $r = 1$ or less, the S-wave solutions are very unsatisfactory. One could speculate that there could be some fundamental difficulties in a theory where fermion masses are larger or equal to the scalar masses or simply that we have not treated the channel $W_L W_L \to F\bar{F}$ in a satisfactory manner.

5   Conclusion

We have made a detailed study of the strongly interacting Higgs with or without the presence of the heavy fourth generation. Without the fourth generation, our conclusion is very much the same as given in the previous study [13]: It is not possible to generate a P-wave resonnance in $W_L W_L$ scattering with a mass less than 3 TeV. A heavy Higgs boson with a mass larger than 3 TeV would generate a lower mass Higgs boson corresponding to the $I = J = 0$ phase shift passing through $90°$ from below which should be considered as as a genuine resonance. The input Higgs boson mass corresponds to a phase shift passing through $90°$ from larger values (antiresonance). Yet a consistent pertubative calculation must be done with the input parameter otherwise the chiral properties are spoiled. Here there is a potential difficulty with the perturbative scheme. It is amusing to note that only in potential scattering theory one can prove that the rate of decrease of the phase shift through $90°$ is related through causality to the range of interaction. The existence of an anti-resonance cannot be excluded a priori.

When the fourth generation is introduced, the problem becomes much more complicated due to the large effective number of fermions. For the ratio of Higgs mass to fermion mass of the order of unity, the P-wave fermion number is positive and is enhanced by a factor of 8, the $I = J = 0$ is enhanced by a factor of 60, the $I = 2$, $J = 0$ is of the order of 2 and they are both negative. Because of the P-wave enhancement factor, the expected mass is much lower than that obtained from the NLSM; furthermore, because the tree $I = J = 1$ amplitude decreases from the low energy theorem value as $(1 - s/m_H^2)$ the P-wave resonance has a much narrower width.

In the $I = J = 0$ channel, because of the enormous negative enhancement factor, the Landau ghost is driven to the origin, which makes the resulting phase shift unacceptable. The same situation holds also for the $I = 2$, $J = 0$ channel. Here we have an even more difficult situation: when the fermion mass is larger or equal to the Higgs mass, a resonance is generated in this channel corresponding to a Landau ghost of the same mass, which is slightly off the real energy axis and on the physical sheet. Unlike the usual result of the screening theorem of Veltman, the presence of the heavy fermion gives rise to a strong dependence on the Higgs mass or more precisely on the ratio of the Higgs boson to the fourth family fermion masses. Our main conclusion is that we can get a reasonable solution with heavy fermions of the order of less than 0.5 Tev provided that the

corresponding Higgs boson is at least twice as heavy. We can also incorporate heavier fermion masses provided we take heavier Higgs mass.

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m_H, TeV	m_f, TeV	$I=0,\ J=0$	$I=1,\ J=1$	$I=2,\ J=0$
0.5	0			
	0.5	0.78	1.11	
	0.7	< 0.78	1.68	
	1	1.14	1.89	1.89
	1.5	0.60	1.68	1.21
	2	0.60	1.37	0.95
0.7	0			
	0.5	0.97	1.13	
	0.7	< 1.2	1.70	
	1	1.20	1.89	1.94
	1.5	0.90	1.70	1.28
	2	0.90	1.41	1.03
1	0			
	0.5		1.17	
	0.7		1.74	
	1		1.89	2.02
	1.5		1.73	1.40
	2		1.48	1.14
1.5	0			
	0.5		1.22	
	0.7		1.89	
	1		1.89	2.19
	1.5		1.80	1.59
	2		1.59	1.32
5	0	1.36		
	0.5	1.27	1.19	1.30
	0.7	1.28	1.69	1.27
	1	1.36	1.96	1.30
	1.5	1.71	2.09	1.57
	2	1.19	1.97	
10	0	1.27		1.25
	0.5	1.21	1.19	1.09
	0.7	1.21	1.65	1.07
	1	1.23	1.98	1.06
	1.5	1.25		1.06
	2	1.31		1.08

Table 1: Scale Λ_c of strong interactions in the higgs sector of the standard model as a function of the mass parameters m_H and m_f ($m_f = 0$ means no fermions). Empty slots mean that Λ_c is far above the inelastic threshold or near the Higgs pole.

m_H, TeV	m_f, TeV	$I = 0$, $J = 0$, TeV^2	$I = 2$, $J = 0$, TeV^2
	1.5	(-0.339, 0.014)	(1.059, 0.163)
1.5	1.	(-0.694, 0.055)	(2.188, 0.659)
	0.7	(-1.017, 0.113)	(3.095, 1.289)
	0.5	(-1.777, 0.321)	(4.752, 3.025)
	1.5	(-0.150, 0.003)	(0.440, 0.030)
	1.	(-0.326, 0.013)	(0.975, 0.139)
1.	0.7	(-0.602, 0.042)	(1.771, 0.439)
	0.5	(-0.898, 0.089)	(2.526, 0.870)
	0.3	(-1.845, 0.345)	
	1.5	(-0.073, 0.001)	(0.209, 0.007)
	1.	(-0.162, 0.003)	(0.469, 0.033)
0.7	0.7	(-0.316, 0.012)	(0.912, 0.122)
	0.5	(-0.553, 0.035)	(1.546, 0.338)
	0.3	(-0.929, 0.095)	
	1.5	(-0.037, 0.000)	(0.104, 0.002)
	1.	(-0.083, 0.001)	(0.235, 0.009)
0.5	0.7	(-0.167, 0.003)	(0.471, 0.034)
	0.5	(-0.308, 0.011)	(0.858, 0.109)
	0.3	(-0.663, 0.050)	
0.2	0.2	(-0.286, 0.010)	(0.740, 0.081)

Table 2: Positions of the complex Landau ghosts as a function of the mass parameters m_H and m_f in the $I = 0$ and $I = 2$ channels using the low-energy expansions of the partial-wave amplitudes.
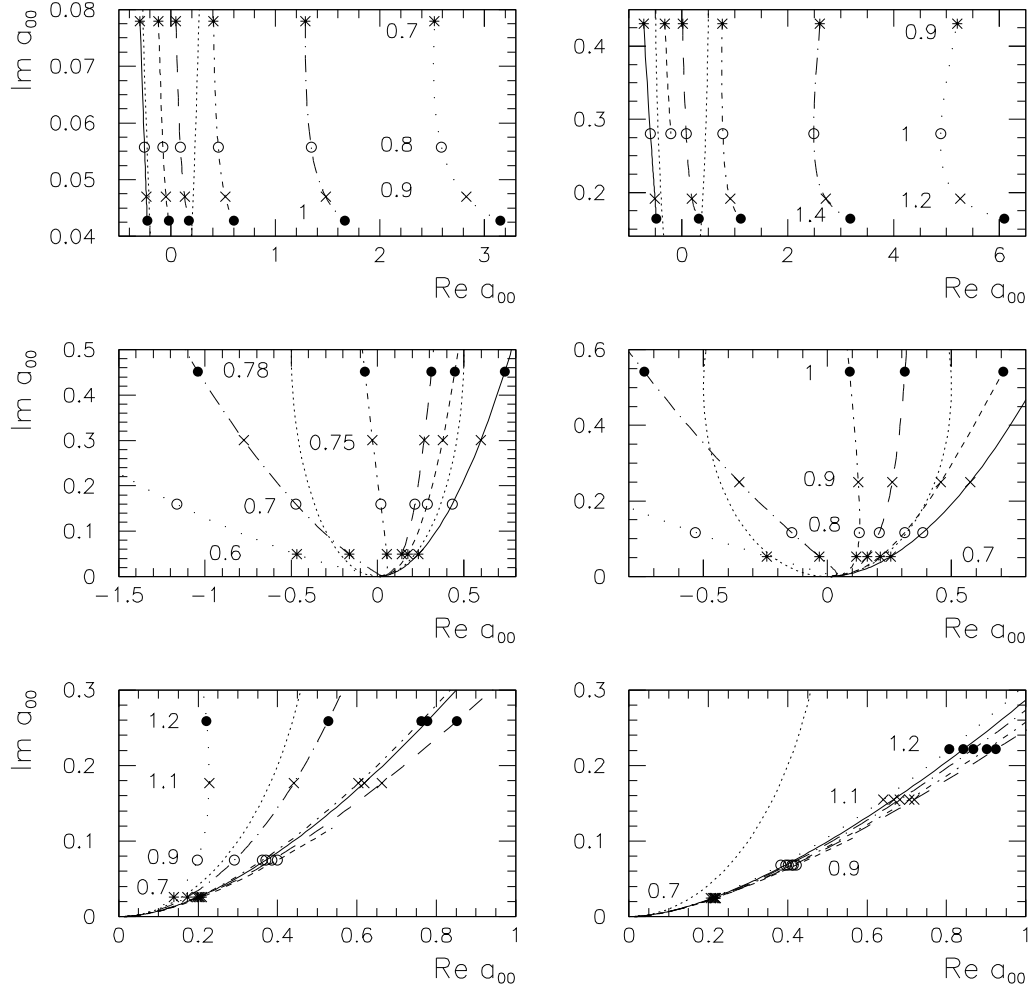
Figure 1: Argand plots for $I = 0$, $J = 0$ channel for $m_H = 0.5$, 0.7, 1, 1.5, 5 and 10 TeV. Solid line corresponds to the case of no fermion contribution, short dashed line to $M_f = 0.5$ TeV, long dashed line to $M_f = 0.7$ TeV, short dash-dotted line to $M_f = 1$ TeV, long dash-dotted line to $M_f = 1.5$ TeV and long dotted line to $M_f = 2$ TeV. Also unitarity circle is shown by short dotted line. Markers denote energy values in TeV.
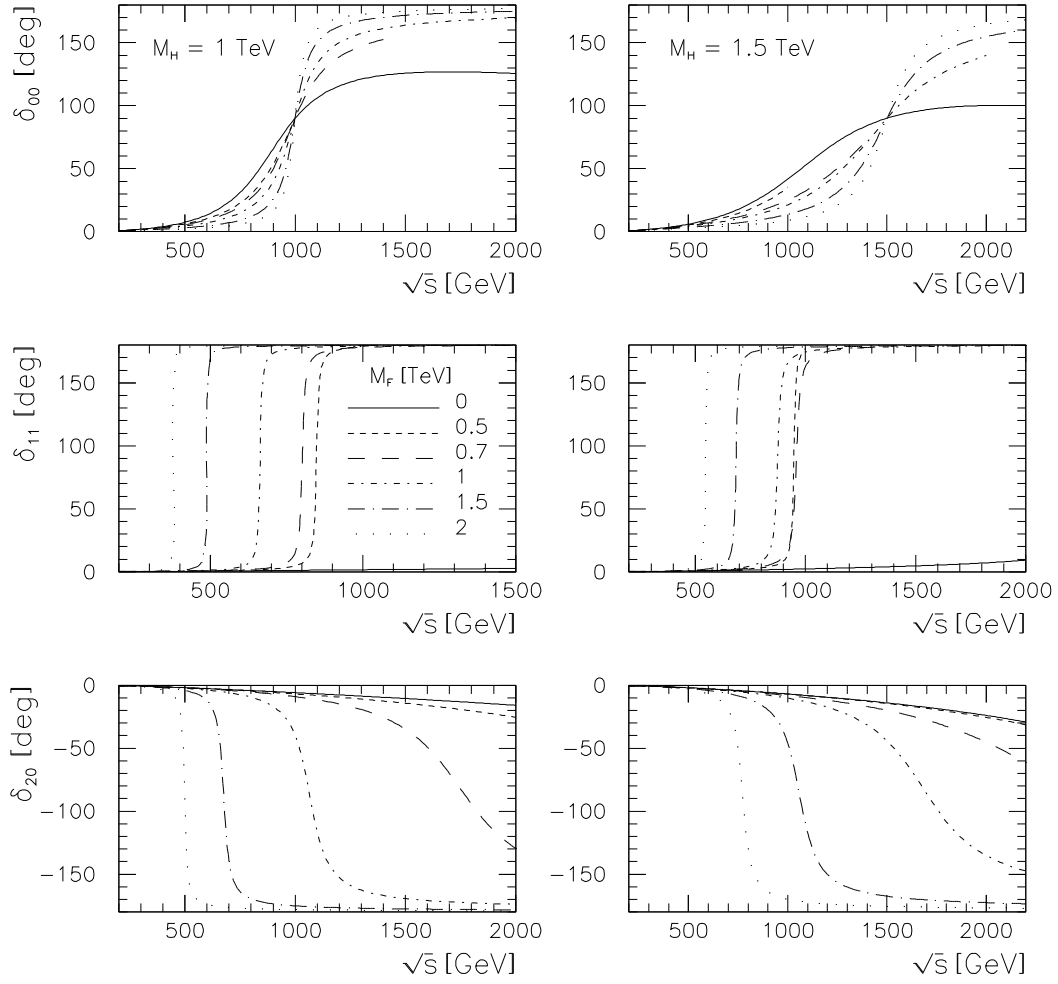
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Figure 2: Phase shifts for $m_H = 1$ and 1.5 TeV as a function of energy for various fermion masses.